Chapter 10

Continuum Neutrosophic Information Geometry. A Differential-Geometric Framework for Triple-Valued Continuous Uncertainty

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ABSTRACT

Information geometry provides a differential–geometric framework for analyzing probability distributions by representing statistical models as manifolds equipped with information-theoretic metrics. However, classical information geometry assumes that uncertainty can be represented by a single probability distribution, which may be insufficient for systems characterized by incomplete, indeterminate, or contradictory information. In this paper we introduce **Continuum Neutrosophic Information Geometry**, a geometric framework built upon continuum neutrosophic probability distributions. In this setting, uncertainty is represented by three density functions corresponding to truth, indeterminacy, and falsity components. These densities define points on a **neutrosophic statistical manifold**, whose geometry is characterized using a generalized Fisher information metric and a neutrosophic divergence extending the Kullback–Leibler divergence. The resulting geometric structure enables the analysis of epistemic uncertainty through concepts such as geodesic trajectories, information distances, and parameter sensitivity. We show that classical information geometry arises as a special case when indeterminacy and falsity components vanish. Potential applications of the framework include machine learning under ambiguous data, sensor fusion with conflicting information sources, Bayesian inference with incomplete evidence, and modeling of complex socio-ecological systems. The proposed theory contributes to the broader development of **continuum neutrosophic mathematics**, integrating logical, probabilistic, statistical, and geometric perspectives on uncertainty.

Keywords: Continuum neutrosophic probability, information geometry, neutrosophic statistical manifolds, Fisher information metric, neutrosophic divergence, epistemic uncertainty, statistical manifolds, uncertainty modeling, information-theoretic geometry, complex systems analysis.

INTRODUCTION

Information geometry provides a powerful mathematical framework for analyzing probability distributions using the tools of differential geometry [1]. In this approach, families of probability distributions are viewed as points on a manifold equipped with geometric structures derived from information-theoretic divergence measures.
This perspective has been applied successfully in many areas, including statistical inference, machine learning, signal processing, and statistical physics.
However, classical information geometry assumes that uncertainty can be represented by a **single probability distribution**. In many real-world systems, uncertainty arises not only from randomness but also from incomplete or contradictory information.

Neutrosophic theory addresses this limitation by representing uncertainty through three independent epistemic components: (*T, I, F*) representing degrees of **truth**, **indeterminacy**, and **falsity** [2, 3].

Recent developments in **continuum neutrosophic probability** extend these components to density functions defined over a continuous refinement index:

$$t(\omega, \alpha), i(\omega, \alpha), f(\omega, \alpha), \alpha \in [0,1].$$

These densities describe the distribution of epistemic evidence across infinitely many latent uncertainty states.

The goal of this paper is to develop a **geometric structure for continuum neutrosophic statistical models**. In particular, we introduce:

- continuum neutrosophic statistical manifolds;
- neutrosophic divergence measures;
- neutrosophic Fisher information metrics;
- geometric interpretations of epistemic uncertainty.

This work connects neutrosophic uncertainty modeling with the broader field of information geometry.

Continuum Neutrosophic Probability Densities

Let (Ω, Σ, P_N) be a **continuum neutrosophic probability space**, where Ω denotes the sample space, Σ is a σ-algebra of measurable events, and P_N represents a neutrosophic probability measure defined on this space.

Unlike classical probability theory, which associates a single scalar probability with each event, the neutrosophic framework represents uncertainty through three independent epistemic components:

$$P_N = (P_T, P_I, P_F)$$

where

- P_T represents the **truth probability measure**, describing epistemic support for the occurrence of events,
- P_I represents the **indeterminacy measure**, capturing unresolved or incomplete information,
- P_F represents the **falsity probability measure**, representing epistemic support against the occurrence of events.

Each component

$$P_T, P_I, P_F : \Sigma \to [0,1]$$

is a classical measure satisfying non-negativity and countable additivity. These three measures collectively describe the full epistemic structure of uncertainty.

To maintain consistency with probabilistic interpretation, the measures satisfy the normalization condition

$$P_T(\Omega) + P_I(\Omega) + P_F(\Omega) = 1.$$

This constraint ensures that the total epistemic mass associated with the sample space remains bounded and interpretable as a generalized probability distribution.

Density Representation

Suppose that the neutrosophic probability measures are **absolutely continuous** with respect to a reference measure λ defined on (Ω, Σ). The reference measure may correspond to Lebesgue measure, counting measure, or another appropriate base measure depending on the application.

By the **Radon–Nikodym theorem** [4, 5, 6], there exist measurable density functions

$$p_T(\omega), p_I(\omega), p_F(\omega)$$

such that

$$P_T(E) = \int_E p_T(\omega)\, d\lambda(\omega)$$

$$P_I(E) = \int_E p_I(\omega)\, d\lambda(\omega)$$

$$P_F(E) = \int_E p_F(\omega)\, d\lambda(\omega)$$

for every event $E \in \Sigma$.

These functions describe the distribution of epistemic evidence across the sample space.

Normalization

The density functions satisfy the global normalization condition

$$\int_{\Omega} (p_T(\omega) + p_I(\omega) + p_F(\omega))\, d\lambda(\omega) = 1.$$

This condition ensures that the total epistemic mass of the system equals one, while allowing the relative contributions of truth, indeterminacy, and falsity to vary across the outcome space.

Neutrosophic Density Triple

The three density functions together form the **neutrosophic probability density triple**

$$p_N(\omega) = (p_T(\omega), p_I(\omega), p_F(\omega)).$$

This triple-valued density provides a richer description of uncertainty than classical probability density functions.

For each outcome $\omega \in \Omega$:

- $p_T(\omega)$ quantifies evidence supporting the occurrence of ω,
- $p_I(\omega)$ represents indeterminate or unresolved epistemic information,
- $p_F(\omega)$ represents evidence opposing the occurrence of ω.

Thus, each outcome is associated not with a single probability value but with a **distribution of epistemic evidence across three dimensions**.

Reduction to Classical Probability

Classical probability theory [7] arises as a special case of this framework when the indeterminacy and falsity components vanish:

$$p_I(\omega) = 0, p_F(\omega) = 0.$$

In this situation the density triple reduces to $p_N(\omega) = (p_T(\omega), 0, 0)$
and the normalization condition becomes $\int_{\Omega} p_T(\omega) d\lambda(\omega) = 1$, which corresponds to the classical probability density function.

Thus, continuum neutrosophic probability densities generalize standard probability distributions while preserving compatibility with classical measure-theoretic probability.

Role in Information Geometry

Within the framework of **continuum neutrosophic information geometry**, the density triple (p_T, p_I, p_F) defines a point in a generalized statistical manifold. The geometric structure of this manifold arises from divergence measures and information metrics defined on these density functions, which will be developed in subsequent sections.
This representation enables the geometric analysis of statistical models in systems characterized by incomplete, uncertain, or contradictory information.

Neutrosophic Statistical Manifolds

Information geometry studies families of probability distributions using the tools of differential geometry by interpreting them as points on a smooth manifold [1]. In the continuum neutrosophic framework, probability distributions are represented not by a single density function but by a **triple of density fields** describing truth, indeterminacy, and falsity components of uncertainty. This structure naturally leads to the concept of a **neutrosophic statistical manifold**.

Let $p_N(\omega \mid \theta)$ denote a parametric family of continuum neutrosophic probability densities defined on the measurable space (Ω, Σ), where $\theta = (\theta_1, \ldots, \theta_k)$ is a vector of parameters belonging to an open subset $\Theta \subseteq \mathbb{R}^k$.
Each neutrosophic density consists of three measurable functions

$$p_N(\omega \mid \theta) = (p_T(\omega \mid \theta),\ p_I(\omega \mid \theta),\ p_F(\omega \mid \theta)).$$

These density functions satisfy the normalization condition

$\int_{\Omega} (p_T(\omega \mid \theta) + p_I(\omega \mid \theta) + p_F(\omega \mid \theta))\, d\omega = 1$, for all parameter values $\theta \in \Theta$.

Definition of the Statistical Manifold

The set $\mathcal{M}_N = \{p_N(\omega \mid \theta) \mid \theta \in \Theta\}$ forms a **continuum neutrosophic statistical manifold**.

Each point of this manifold represents a neutrosophic probability distribution, and the parameter vector θserves as a coordinate chart on the manifold. The dimension of the manifold equals the number of parameters k.
Unlike classical statistical manifolds, where each point corresponds to a single probability density function, a point in the neutrosophic statistical manifold represents a **triple density structure** (p_T, p_I, p_F) capturing different epistemic aspects of uncertainty.

Tangent Space

To analyze the local geometry of the statistical manifold, we consider infinitesimal variations of the density triple with respect to the parameters.
The tangent vectors at a point $p_N(\omega \mid \theta)$are defined by the partial derivatives

$$\frac{\partial}{\partial \theta_i} p_N(\omega \mid \theta) = \left(\frac{\partial p_T}{\partial \theta_i}, \frac{\partial p_I}{\partial \theta_i}, \frac{\partial p_F}{\partial \theta_i}\right).$$

These vectors span the **tangent space** $T_\theta \mathcal{M}_N$ of the neutrosophic statistical manifold at parameter value θ.
The tangent space therefore consists of triples of functions representing infinitesimal changes in the truth, indeterminacy, and falsity density components.

Log-Density Representation

In classical information geometry, many geometric quantities are expressed using the **log-likelihood function** [8].
The same idea can be extended to the neutrosophic framework.
Define the log-density components

$$\ell_T(\omega \mid \theta) = \log\, p_T(\omega \mid \theta),$$
$$\ell_I(\omega \mid \theta) = \log\, p_I(\omega \mid \theta),$$
$$\ell_F(\omega \mid \theta) = \log\, p_F(\omega \mid \theta).$$

The gradients $\frac{\partial \ell_T}{\partial \theta_i}, \frac{\partial \ell_I}{\partial \theta_i}, \frac{\partial \ell_F}{\partial \theta_i}$ describe how the epistemic densities change with respect to the parameters.
These quantities play a central role in defining the **information metric** of the neutrosophic statistical manifold.

Geometric Interpretation

Geometrically, each neutrosophic probability distribution corresponds to a point in a functional space composed of three coupled density functions (see Figure 1) . The statistical manifold can therefore be interpreted as a **three-layer information geometry**, where each layer corresponds to one epistemic component.
More precisely:

- the **truth layer** represents distributions describing positive epistemic evidence,
- the **indeterminacy layer** captures unresolved uncertainty,
- the **falsity layer** represents opposing evidence.

The geometry of the manifold describes how these three components evolve as model parameters change.
This structure generalizes classical statistical manifolds by incorporating additional epistemic dimensions while preserving the underlying geometric framework of information theory [1].

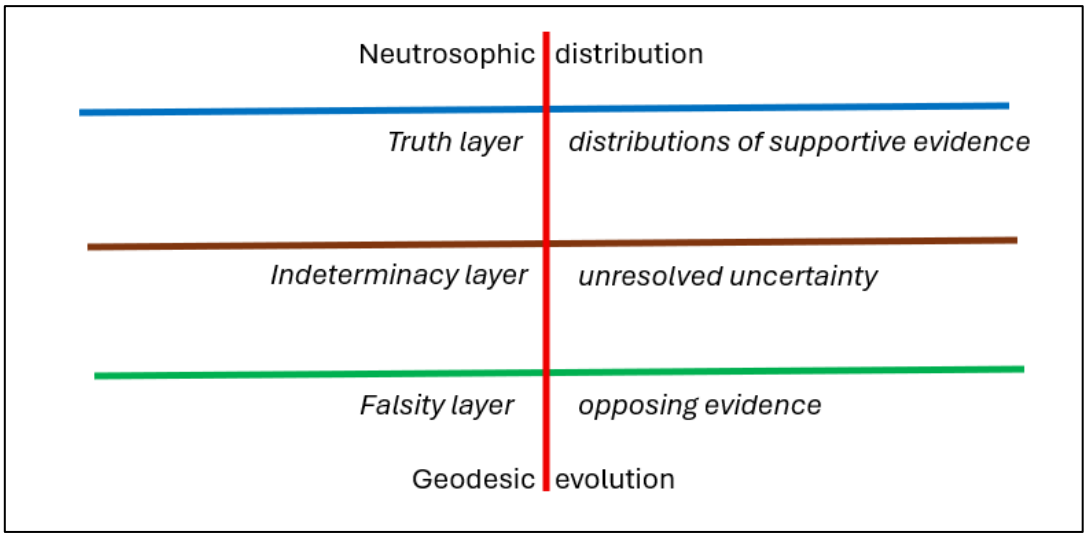

Figure 1. Geometric interpretation of the continuum neutrosophic statistical manifold.

Each neutrosophic probability distribution corresponds to a point represented by a triple of coupled density functions (p_T, p_I, p_F). The manifold can be viewed as a three-layer information geometry composed of truth, indeterminacy, and falsity layers. Geodesic trajectories on the manifold describe the evolution of epistemic states as model parameters change.

Reduction to Classical Statistical Manifolds

The neutrosophic statistical manifold reduces to the classical statistical manifold when the indeterminacy and falsity components vanish: $p_I(\omega \mid \theta) = 0, p_F(\omega \mid \theta) = 0$.
In this case the density triple reduces to $p_N(\omega \mid \theta) = (p_T(\omega \mid \theta), 0, 0)$ and the manifold $\mathcal{M}_N$ becomes the classical manifold of probability distributions studied in information geometry.
Thus, continuum neutrosophic information geometry extends the classical theory while maintaining compatibility with existing statistical frameworks.

Role in Neutrosophic Information Geometry

The neutrosophic statistical manifold provides the geometric foundation for analyzing the structure of continuum neutrosophic probability distributions. In subsequent sections we will define divergence measures and information metrics on this manifold, allowing the study of geodesic curves, statistical inference, and the evolution of uncertainty within the neutrosophic framework.

Neutrosophic Fisher Information Metric and Dual Affine Connections

A central concept in information geometry is the **Fisher information metric**, which defines a Riemannian metric on the statistical manifold of probability distributions [1, 8]. This metric measures the sensitivity of probability distributions to changes in model parameters and plays a fundamental role in statistical inference.
In the continuum neutrosophic framework, probability distributions are represented by **triples of density functions** corresponding to truth, indeterminacy, and falsity components. Consequently, the information metric must account for contributions from all three epistemic layers.

Neutrosophic Fisher Information Metric

Let $p_N(\omega \mid \theta) = (p_T(\omega \mid \theta), p_I(\omega \mid \theta), p_F(\omega \mid \theta))$ be a parametric family of continuum neutrosophic densities defined on the statistical manifold $\mathcal{M}_N$.

The **neutrosophic Fisher information metric** is defined as $g_{ij}^{(N)}(\theta) = g_{ij}^{(T)}(\theta) + g_{ij}^{(I)}(\theta) + g_{ij}^{(F)}(\theta)$, where each term corresponds to the Fisher information associated with one epistemic component.

The truth component is given by $g_{ij}^{(T)}(\theta) = \int_{\Omega} \frac{\partial \log\, p_T(\omega|\theta)}{\partial \theta_i} \frac{\partial \log\, p_T(\omega|\theta)}{\partial \theta_j} p_T(\omega \mid \theta)\, d\omega$

with analogous expressions for the indeterminacy and falsity components:

$$g_{ij}^{(I)}(\theta) = \int_{\Omega} \frac{\partial \log\, p_I(\omega \mid \theta)}{\partial \theta_i} \frac{\partial \log\, p_I(\omega \mid \theta)}{\partial \theta_j} p_I(\omega \mid \theta)\, d\omega$$

$$g_{ij}^{(F)}(\theta) = \int_{\Omega} \frac{\partial \log\, p_F(\omega \mid \theta)}{\partial \theta_i} \frac{\partial \log\, p_F(\omega \mid \theta)}{\partial \theta_j} p_F(\omega \mid \theta)\, d\omega.$$

Thus, the neutrosophic Fisher metric is obtained by summing the Fisher metrics of the three epistemic components.

Interpretation

The neutrosophic Fisher metric quantifies how rapidly the epistemic structure of the probability distribution changes with respect to parameter variations.
More specifically:

- the **truth information** component measures sensitivity of supportive evidence,
- the **indeterminacy information** component measures sensitivity of unresolved uncertainty,
- the **falsity information** component measures sensitivity of contradictory evidence.

Together these components define the **local geometry of the neutrosophic statistical manifold**.

Dual Affine Connections

In classical information geometry, the statistical manifold possesses a pair of dual affine connections associated with exponential and mixture coordinate systems [1].
These connections can be generalized to the neutrosophic setting.
Let $\nabla^{(e)}, \nabla^{(m)}$ denote the **exponential** and **mixture** connections defined on the neutrosophic statistical manifold.
For the truth component, the exponential connection coefficients are defined by $\Gamma_{ijk}^{(e,T)} = \int_{\Omega} \frac{\partial^2 \log\, p_T}{\partial \theta_i \partial \theta_j} \frac{\partial \log\, p_T}{\partial \theta_k} p_T\, d\omega.$

Analogous expressions hold for the indeterminacy and falsity components.
The global neutrosophic connection coefficients are obtained by summing these contributions:

$$\Gamma_{ijk}^{(e,N)} = \Gamma_{ijk}^{(e,T)} + \Gamma_{ijk}^{(e,I)} + \Gamma_{ijk}^{(e,F)}.$$

A similar construction applies to the mixture connection $\nabla^{(m)}$.

Geometric Structure

The statistical manifold $\left(\mathcal{M}_N, g^{(N)}, \nabla^{(e)}, \nabla^{(m)}\right)$ thus forms a **dualistic geometric structure**, extending the classical theory of statistical manifolds to neutrosophic probability distributions.
This structure enables the analysis of:

- geodesic paths between neutrosophic distributions,
- information flow in epistemic systems,
- optimal statistical estimators under triple-valued uncertainty.

Reduction to Classical Fisher Geometry

If the indeterminacy and falsity components vanish, $p_I = 0, p_F = 0$, the neutrosophic Fisher metric reduces to the classical Fisher information metric $g_{ij} = \int_\Omega \frac{\partial \log p}{\partial \theta_i} \frac{\partial \log p}{\partial \theta_j} p(\omega)\, d\omega$
studied in information geometry [8].
Thus, classical information geometry appears as a special case of the neutrosophic framework.

Neutrosophic Divergence and Information Distance

A central concept in information geometry is the use of **divergence functions** to measure the difference between probability distributions. In classical information theory, the most widely used divergence is the **Kullback–Leibler (KL) divergence** [9], which quantifies the information loss incurred when one distribution is used to approximate another [10]. Divergence functions play a fundamental role in defining the geometric structure of statistical manifolds because they generate both the Fisher information metric and the associated dual affine connections [1]. In the continuum neutrosophic framework, probability distributions are represented by triples of density functions describing truth, indeterminacy, and falsity components. Consequently, divergence between neutrosophic distributions must account for differences across all three epistemic dimensions.

Definition of Neutrosophic Divergence

Let $P_N = (p_T, p_I, p_F)$ and $Q_N = (q_T, q_I, q_F)$ be two continuum neutrosophic probability distributions defined on the same measurable space.
The **neutrosophic divergence** between P_N and Q_N is defined as $D_N(P_N \parallel Q_N) = D_T + D_I + D_F$, where each component measures the divergence between the corresponding density functions.
The truth divergence is defined as $D_T = \int_\Omega p_T\,(\omega) \log \frac{p_T(\omega)}{q_T(\omega)}\, d\omega$ with analogous expressions for the indeterminacy and falsity components:

$$D_I = \int_\Omega p_I\,(\omega) \log \frac{p_I(\omega)}{q_I(\omega)}\, d\omega$$

$$D_F = \int_\Omega p_F\,(\omega) \log \frac{p_F(\omega)}{q_F(\omega)}\, d\omega.$$

Thus, the neutrosophic divergence is obtained by summing the divergences of the three epistemic components.

Properties of Neutrosophic Divergence

The neutrosophic divergence inherits several important properties from the classical Kullback–Leibler divergence [9].

Non-negativity $D_N(P_N \parallel Q_N) \geq 0$ *for all neutrosophic distributions* P_N *and* Q_N.

Identity of indiscernibles $D_N(P_N \parallel Q_N) = 0$
if and only if $p_T = q_T, p_I = q_I, p_F = q_F$.

Asymmetry

In general $D_N(P_N \parallel Q_N) \neq D_N(Q_N \parallel P_N)$ which reflects the directional nature of information divergence.
These properties make neutrosophic divergence a natural generalization of classical information-theoretic distance measures.

Relation to the Fisher Information Metric

In information geometry, the Fisher information metric arises as the second-order approximation of the divergence between nearby distributions [8].
Let $P_N(\theta)$ be a parametric neutrosophic distribution and consider a nearby distribution
$P_N(\theta + d\theta)$.

Expanding the neutrosophic divergence to second order yields $D_N(P_N(\theta) \parallel P_N(\theta + d\theta)) = \frac{1}{2} g_{ij}^{(N)} d\theta_i d\theta_j + O(\| d\theta \|^3)$, where $g_{ij}^{(N)}$ is the neutrosophic Fisher information metric.

Thus, the divergence function generates the Riemannian geometry of the neutrosophic statistical manifold.

Symmetrized Information Distance

For certain applications it is convenient to define a symmetric divergence measure.

The **symmetrized neutrosophic divergence** may be defined as

$$D_N^{(sym)}(P, Q) = \frac{1}{2}[D_N(P \parallel Q) + D_N(Q \parallel P)].$$

This quantity provides a symmetric measure of information distance between neutrosophic probability distributions.

Reduction to Classical Divergence

When the indeterminacy and falsity components vanish, $p_I = q_I = 0, p_F = q_F = 0$, the neutrosophic divergence reduces to the classical Kullback–Leibler divergence

$$D(P \parallel Q) = \int p(\omega) \log \frac{p(\omega)}{q(\omega)} \, d\omega.$$

Thus, classical information geometry appears as a special case of the neutrosophic divergence framework.

Geometric Interpretation

Within the neutrosophic statistical manifold, the divergence function defines the **information distance** between two epistemic states. Each state corresponds to a triple density distribution describing supportive, indeterminate, and contradictory evidence.

The divergence therefore measures how strongly the epistemic structure of one distribution differs from another. In this sense, the neutrosophic divergence provides a geometric description of the **evolution of uncertainty** across the statistical manifold.

Geodesics and Epistemic Evolution

In information geometry, the geometry of a statistical manifold provides insight into how probability distributions evolve under parameter changes. In particular, **geodesic curves** describe the shortest or most natural paths between distributions on the manifold [1, 8]. These curves play an important role in statistical inference, learning algorithms, and information-theoretic analysis.

Within the continuum neutrosophic framework, probability distributions are represented by triples of density functions corresponding to truth, indeterminacy, and falsity components. Consequently, geodesic curves describe the smooth evolution of these epistemic components across the statistical manifold.

Geodesic Curves on the Neutrosophic Statistical Manifold

Let $p_N(\omega \mid \theta) = (p_T(\omega \mid \theta), p_I(\omega \mid \theta), p_F(\omega \mid \theta))$ be a parametric family of continuum neutrosophic distributions defined on the statistical manifold $\mathcal{M}_N$.

A **geodesic curve** on the manifold is a path $\theta(t), t \in [0,1]$ that satisfies the geodesic equation associated with the neutrosophic Fisher metric $g_{ij}^{(N)}$.

The geodesic equation takes the form

$\frac{d^2\theta^k}{dt^2} + \Gamma_{ij}^k \frac{d\theta^i}{dt} \frac{d\theta^j}{dt} = 0$, where Γ_{ij}^k are the Christoffel symbols associated with the neutrosophic Fisher metric.

This equation determines the trajectory along which the neutrosophic probability distribution evolves most naturally within the statistical manifold.

Evolution of Epistemic Components

Along a geodesic path, the three density components evolve simultaneously:

$$p_T(\omega \mid \theta(t)), p_I(\omega \mid \theta(t)), p_F(\omega \mid \theta(t)).$$

This evolution can be interpreted as a continuous transformation of the epistemic state of the system.
More specifically:

- changes in p_T describe variation in **supportive evidence**,
- changes in p_I represent evolution of **indeterminate information**,
- changes in p_F reflect variation in **contradictory evidence**.

The combined evolution of these three components defines a trajectory in the neutrosophic statistical manifold.

Exponential and Mixture Geodesics

Information geometry distinguishes between two important types of geodesics corresponding to the dual affine connections introduced earlier [1]:

Exponential Geodesics

These correspond to linear paths in exponential coordinates and are typically associated with parametric statistical models.

Mixture Geodesics

These correspond to linear interpolation between probability distributions.
In the neutrosophic framework, mixture geodesics may be written as: $p_N^{(t)} = (1-t)p_N^{(1)} + tp_N^{(2)}$, where $p_N^{(1)}$ and $p_N^{(2)}$ are two neutrosophic distributions.
This representation corresponds to gradual interpolation between two epistemic states.

Geometric Interpretation of Uncertainty Evolution

Geodesic paths provide a geometric description of how uncertainty evolves as information is updated or model parameters change.
For example:

- **learning processes** correspond to trajectories on the statistical manifold,
- **Bayesian updates** can be interpreted as movement toward regions of lower divergence,
- **information fusion** corresponds to combining epistemic components along mixture geodesics.

In this sense, the neutrosophic statistical manifold provides a geometric framework for analyzing the dynamics of epistemic uncertainty.

Reduction to Classical Information Geometry

If the indeterminacy and falsity components vanish, $p_I = 0, p_F = 0$, the neutrosophic statistical manifold reduces to the classical statistical manifold studied in information geometry.

In this case the geodesic equation reduces to the standard Fisher–Rao geodesics governing classical probability distributions [8].

Thus, classical information geometry appears as a special case of the neutrosophic geometric framework.

Applications of Continuum Neutrosophic Information Geometry

The geometric framework developed in the previous sections provides new analytical tools for studying systems characterized by complex uncertainty structures. By representing neutrosophic probability distributions as points on a statistical manifold equipped with an information metric and divergence structure, continuum neutrosophic information geometry allows the analysis of epistemic uncertainty using geometric concepts such as distance, curvature, and geodesic trajectories.
Several application domains can benefit from this approach.

Machine Learning and Statistical Learning

Information geometry has played an important role in machine learning by providing geometric interpretations of learning algorithms, optimization procedures, and parameter estimation [4]. Gradient-based learning methods, natural gradient descent, and variational inference all rely on geometric properties of statistical manifolds.
In many machine learning problems, training data may contain ambiguous labels, missing information, or contradictory observations. Classical probabilistic models often struggle to represent these situations because they rely on a single probability distribution.
Continuum neutrosophic information geometry allows machine learning models to represent uncertainty using three epistemic components. Learning algorithms may then be interpreted as trajectories on the neutrosophic statistical manifold that minimize divergence between model predictions and observed data. The neutrosophic Fisher metric can also be used to define **natural gradient learning algorithms** adapted to triple-valued uncertainty structures.

Sensor Fusion and Information Integration

Modern sensor networks often combine data from multiple sources with varying levels of reliability. Measurements from different sensors may contain noise, partial information, or contradictory signals.

Neutrosophic information geometry provides a natural framework for analyzing such systems. Each sensor observation can be represented as a neutrosophic probability distribution describing supportive evidence, indeterminate information, and contradictory evidence. Information fusion can then be interpreted as moving along geodesic paths on the neutrosophic statistical manifold that minimize divergence between multiple information sources.

This geometric interpretation complements existing information fusion frameworks such as the Dezert–Smarandache theory of evidence [11].

Bayesian Inference

Bayesian inference describes how probability distributions evolve when new information becomes available. In information geometry, Bayesian updates can be interpreted as movements on the statistical manifold that minimize information divergence between prior and posterior distributions [1].

Within the continuum neutrosophic framework, prior and posterior beliefs may be represented by neutrosophic distributions. The updating process then corresponds to a trajectory on the neutrosophic statistical manifold reflecting the incorporation of new evidence.

This approach is particularly useful in situations where observations are incomplete or conflicting, since indeterminacy and falsity components can explicitly represent unresolved information.

Complex Systems and Socio-Ecological Modeling

Many real-world systems involve complex interactions between natural processes, human decision-making, and uncertain observations. Examples include ecological monitoring, climate systems, and socio-economic networks. Such systems frequently involve multiple layers of uncertainty, including stochastic variability, incomplete knowledge, and conflicting data sources. Continuum neutrosophic information geometry provides a framework for analyzing these systems by modeling the evolution of epistemic uncertainty on a statistical manifold.
Geometric quantities such as divergence and curvature may help identify regions of high uncertainty or instability in complex systems.

Information-Theoretic Analysis of Uncertainty

Entropy and divergence measures are widely used to analyze the information content of statistical models [10]. In the neutrosophic framework, these measures can be extended to capture the contributions of truth, indeterminacy, and falsity components.
The geometric interpretation provided by neutrosophic information geometry enables the study of **information flow, uncertainty propagation, and learning dynamics** in systems characterized by multi-dimensional uncertainty structures.

Conclusion

This paper introduced **Continuum Neutrosophic Information Geometry**, a differential-geometric framework for analyzing probability distributions characterized by triple-valued epistemic uncertainty. By extending classical information geometry to the neutrosophic setting, we developed a geometric structure capable of representing uncertainty that includes not only stochastic variability but also indeterminacy and contradictory information.

The study established the fundamental elements of this framework. First, continuum neutrosophic probability densities were defined as triples of density functions corresponding to truth, indeterminacy, and falsity components. These densities were used to construct **neutrosophic statistical manifolds**, whose points represent continuum neutrosophic probability distributions.

Second, the paper introduced the **neutrosophic Fisher information metric**, which generalizes the classical Fisher metric by incorporating contributions from all three epistemic components. This metric defines the local geometry of the neutrosophic statistical manifold and provides a measure of the sensitivity of neutrosophic probability distributions to parameter variations.

Third, a **neutrosophic divergence measure** was defined as an extension of the Kullback–Leibler divergence. This divergence generates the information metric and provides a natural measure of distance between neutrosophic distributions. Using this structure, geodesic curves were introduced to describe the smooth evolution of epistemic states on the statistical manifold.

Together, these elements establish a geometric framework for analyzing the structure and dynamics of continuum neutrosophic probability distributions. The resulting theory extends classical information geometry while preserving compatibility with established principles of statistical inference and information theory[1, 8].

The framework also opens new possibilities for analyzing complex systems characterized by incomplete, ambiguous, or conflicting information. Potential applications include machine learning, sensor fusion, Bayesian inference, and the modeling of socio-ecological systems where multiple layers of uncertainty coexist.

From a broader perspective, the present work contributes to the development of a **continuum neutrosophic mathematical program** that integrates logical, probabilistic, statistical, and geometric aspects of uncertainty modeling. Previous studies introduced continuum neutrosophic sets, probability measures, and statistical models. The geometric perspective developed here provides a unifying structure that links these elements within a coherent theoretical framework.

Future research may explore several promising directions, including the study of curvature properties of neutrosophic statistical manifolds, the development of natural gradient algorithms for neutrosophic learning systems, and applications in large-scale data analysis where uncertainty cannot be adequately represented by single probability distributions.

In summary, **continuum neutrosophic information geometry** provides a new mathematical perspective on uncertainty, combining concepts from neutrosophic theory, probability, and information geometry. This framework may serve as a foundation for further theoretical developments and practical applications in the analysis of complex uncertain systems.

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